Adjacent Structures Connected by Viscous Damper Subjected to Random Excitation

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Abstract—**Dynamic response of two adjacent single degree-of-freedom (SDOF) structures connected by viscous damper under base excitation is investigated. The base excitation is modeled as a stationary white-noise random process. The equations of motion of coupled structures are derived and mean square responses are obtained. The response parameters considered are relative displacement and absolute acceleration. It is observed that by increasing the damping coefficient of damper the mean square displacement response and mean square acceleration response are decreases and after certain value it again increases. There is some optimum value of damping coefficient of damper for which mean square response value attain minimum. A parametric study is also carried out to investigate the effect of system parameters on optimum damping coefficient of damper and corresponding responses. The system parameters considered are mass ratio and frequency ratio. It is found that viscous damper is quite effective for response control of coupled structures subjected to random excitation. It has been observed that the frequency ratio has significant effect on the response control of the coupled structure; where as the effect of mass ratio is marginal.**

Key words: Adjacent structures, Mean square response, Optimum damping, Viscous damper.

I. Introduction

The natural disturbances like strong wind and earthquakes produce excessive structural vibrations, which creates human discomfort and many times lead to catastrophic structural failure as well. In last decade, significant efforts have been given for design of engineering structures with various control strategies to increase their safety and reliability against strong external excitations. Many energy dissipation devices and control system have been developed to reduce the excessive structural vibrations due to natural disturbances. These control strategies are able to modify dynamically the response of structure in a desirable manner, thereby termed as protective systems for the new structures and the existing structures can be retrofitted or strengthened effectively to withstand future natural disturbances. The control system and the structure do not behave as independent dynamic system but rather interact with each other. In addition, the interaction effects also occur between the excitation and structure (i.e. soil-structure interaction). According to the energy consumptions, control system can be classified as active, passive, semi-active and hybrid control system.

Active control system may be defined as a system, which provide additional energy to the controlled structure and opposite to that delivered by the dynamic loading. The control forces are supplied to the structure by means of electro-hydraulic or electro-mechanical actuators, which require a large power source for their operation. Control forces are developed based on the feedback from sensors that measure the excitation and/or the response of the structure. The feedback from the structural response may be measured at locations remote from the location of the active control system. As a result, active control mechanisms are more complex mechanism, requiring sensors and evaluator/controller equipments. Cost and maintenance of such system is also significantly higher than that of passive devices. Various active control mechanisms are active mass/tuned mass dampers, active tendon system, and active tuned liquid damper.

Passive control system may be defined as a system, which does not require an external power source for operation and utilizes the response of the structure to develop the control forces. Control forces are developed at the location of the passive control system, as a function of the response of the structure. The various passive vibration control mechanisms are viscous dampers, visco-elastic dampers, friction dampers, tuned mass dampers, tuned liquid dampers, metallic yielding dampers, and base isolators. More complete details on the mechanics and working principles of these devices can be found in reference [1].

Semi-active control system may be defined as a system which is combination of passive and active control mechanism. Semi-active control system require a small external power source for operation (e.g. a battery) and utilize the motion of the structure to develop the control force, the magnitude of which can be adjusted by external power source. The advantage is that in case of power failure the passive component of the control will still offer some protection. Control forces are developed based on feedback from sensors that measure the excitation and/or the response of the structure. The feedback from the structural response may be measured at locations remote from the location of the semi-active control system. The various semi-active mechanisms are variable orifice dampers, variable friction dampers, variable stiffness dampers, controllable tuned liquid dampers, controllable fluid dampers (Electorheological (ER) and Megnetorheological (MR)) dampers.

Hybrid control system consists of combination of passive and active devices or combined passive and semi-active devices. Because multiple control devices are operating, hybrid control system can alleviate some of the restriction and limitations that exist when each system is acting alone. Thus, higher level of performance may be achievable. An additional benefit of hybrid system is that, in the case of power failure, the passive components of the control still offer some of protection. The various hybrid mechanisms are hybrid mass dampers and hybrid base isolation.

Because of limited availability of land and preference for centralized services, buildings in a modern city are often built closely to each other. Some tall buildings are often built with podium structures to achieve a large open space for parking shops, restaurants, and hotel lobbies at the ground level. If the separation between adjacent structures is not sufficient, or if they are not separated with proper structural connections mutual pounding may also occur during an earthquake, which has been observed during Loma Prieta earthquake [2] and many more past earthquake events.

The concept of linking adjacent structures using passive dampers, active dampers and semi-active dampers has thus been proposed to improve their dynamic performance. Using special energy dissipation devices of appropriate capacity and at proper position, the energy dissipation capacity of the adjacent structures can be increased. The concept is to allow two dynamically dissimilar structures to exert control force upon one another. It improves the performance of the system by reducing overall response of the system. Moreover, it also overcomes the problem of pounding, which is more sever-load condition than the case of the vibration without pounding. The dynamic response of adjacent structures connected by friction damper has been investigated [3]. The seismic response of dynamically similar adjacent building connected with viscous dampers have been investigated [4]

In this paper the adjacent structures connected with viscous damper under random excitation is studied, and the effectiveness of the viscous damper for random response reduction of adjacent structure is investigated.

II. MODELING OF CONNECTED STRUCTURES

Let us consider two adjacent structures connected with a viscous damper as shown n Fig.1. The adjacent structures are idealized as SDOF systems and referred as Structure1 and 2. The two structures are assumed to be symmetric with their symmetric planes in alignment. The ground motion is assumed to occur in one direction in the symmetric planes of the structures so that the problem can be simplified as a two-dimensional problem as shown in Fig.1. Both the structures are assumed to be supported on stiff ground and subjected to the same ground acceleration. The viscous damper is modeled as linear dash pot, in which the force is proportional to the relative velocity of its both ends. The corresponding mathematical model of the damper connected structure is shown in Fig. 2. Let m_1, c_1, k_1 and m_2, c_2, k_2 be the mass, damping coefficient and stiffness of the Structure 1 and 2, respectively. The natural frequency of the structure is given by $\omega_1 = \sqrt{k_1/m_1}$ and $\omega_2 = \sqrt{k_2/m_2}$ for Structure 1 and 2, respectively. The damping ratio of Structure 1 and 2, is given by $\xi_1 = c_1/2m_1\omega_1$ and $\xi_2 = c_2/2m_2\omega_2$, respectively. Let β and μ be the frequency and mass ratio of two structures defined as

$$\mu = \frac{m_1}{m_2} \tag{1}$$

$$\beta = \frac{\omega_1}{\omega_2} \tag{2}$$

Let c_d be the damping coefficient of the damper, which is expressed in the mathematical form as

$$\xi_d = \frac{c_d}{2m_1\omega_1} \tag{3}$$

where ξ_d is the normalized damping coefficient of damper.

The governing equations of motion for the damper connected system can be written as

$$m_1 x_1 + c_1 x_1 + k_1 x_1 + c_d(x_1 - x_2) = -m_1 x_g \tag{4}$$

$$m_2 x_2 + c_2 x_2 + k_2 x_2 - c_d(x_1 - x_2) = -m_2 x_g \tag{5}$$

where x_1 and x_2 are the displacement responses, relative to the ground of Structures 1 and 2, respectively; and x_g is the ground acceleration.

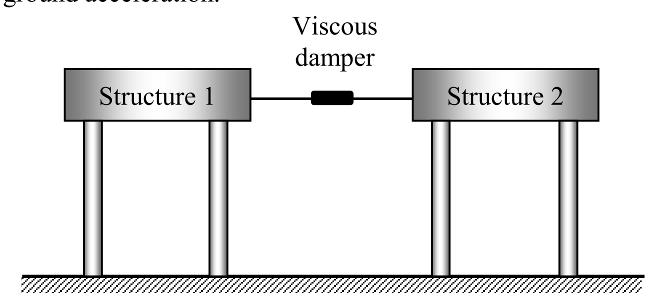

Fig. 1 Adjacent Structures with viscous damper

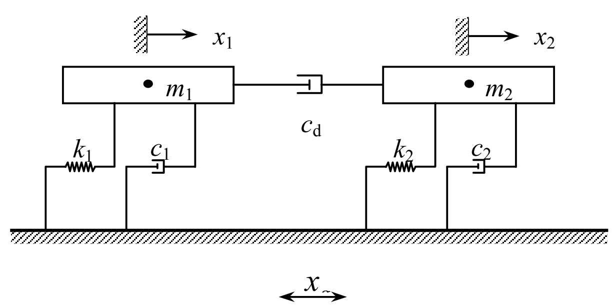

Fig. 2 Mathematical model

III. RESPONSE TO STATIONARY WHITE-NOISE RANDOM EXCITATION

The analytical expressions for mean square displacement and mean square acceleration responses of the damper connected system are derived. The structural control criteria depend on the nature of dynamic loads and the response quantities of interest. Minimizing the relative displacement or absolute acceleration of the system has always been considered as the control objective. In case of flexible structures, displacement are predominant that need to be controlled. On contrary to this, in case of stiff structures, accelerations are of more concern generating higher inertia forces in structures, which should be mitigated. Let the coupled system subjected to the base acceleration x_g, modeled as Gaussian white-noise random process with constant power spectral density S_0. The mean square displacement, $\sigma_{x_1}^2$ and $\sigma_{x_2}^2$ of Structures 1 and 2, respectively are expressed as [5]

$$\sigma_{x_1}^2 = \int_{-\infty}^{\infty} \left| x_1(i\omega) \right|^2 S_0 \, d\omega \tag{6}$$

$$\sigma_{x_2}^2 = \int_{-\infty}^{\infty} \left| x_2(i\omega) \right|^2 S_0 \, d\omega \tag{7}$$

where $x_1(i\omega)$ and $x_2(i\omega)$ are the harmonic transfer function for displacement responses, x_1 and x_2 respectively and are expressed by

$$x_1(i\omega) \frac{\omega^2 - \omega_2^2 - i\omega(\Delta_d + \Delta_2)}{\begin{array}{l}\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) \\ +\omega_1^2\omega_2^2 - i\omega^3(\Delta_1 + \Delta_2 + \Delta_d) \\ +i\omega(\omega_1^2\Delta_2 + \omega_1^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}} \tag{8}$$

$$x_2(i\omega) \frac{\omega^2 - \omega_1^2 - i\omega(\Delta_d + \Delta_1)}{\begin{array}{l}\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) \\ +\omega_1^2\omega_2^2 - i\omega^3(\Delta_1 + \Delta_2 + \Delta_d) \\ +i\omega(\omega_1^2\Delta_2 + \omega_1^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}} \tag{9}$$

by integrating Eqs. (6) and (7), considering equal damping ratio in both the structures $\xi_1 = \xi_2 = \xi$, the mean square displacement, $\sigma_{x_1}^2$ and $\sigma_{x_2}^2$ of Structures 1 and 2, respectively are given as

$$\sigma_{x1}^2 = \frac{2\pi S_0}{\omega_1^3}\left[\frac{\begin{array}{l}\beta(1+\beta)^2\xi((-1+\beta)^2 + 4\beta\xi^2) \\ +(1+\beta)((-1+\beta)^2(1+\beta)\mu \\ +4\beta(2+\beta+\beta^2+2(1+\beta)\mu)\xi^2)\xi_e \\ +4(1+\mu)(1+\mu+\beta(3+\beta^2 \\ +(3+\beta)\mu))\xi\xi_e^2 \\ +4(1+\mu)^3\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2 + 4\beta\xi^2) \\ +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2 + 8\beta\xi^2)\xi_e \\ +((1+\beta^2)^2\mu + 4(\beta^4 + \mu^2 + 3\beta^3(1+\mu) \\ +3\beta\mu(1+\mu) + \beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2 \\ +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{10}$$

$$\sigma_{x2}^2 = \frac{2\pi S_0}{\omega_2^3}\left[\frac{\begin{array}{l}\beta^2(4\beta(1+\beta)^2\xi^3 + \xi_e \\ +4(1+\beta)(\mu+\beta(2+\mu+2\beta(1+\mu)))\xi^2\xi_e \\ +\beta^2\xi_e(-2+\beta^2+4(1+\mu)^3\xi_e^2) \\ +\xi((-1+\beta^2)^2 + 4(1+\mu) \\ (\mu+\beta(1+\beta(3+\beta)(1+\mu)))\xi_e^2))\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2 + 4\beta\xi^2) \\ +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2 + 8\beta\xi^2)\xi_e \\ +((1+\beta^2)^2\mu + 4(\beta^4 + \mu^2 + 3\beta^3(1+\mu) \\ +3\beta\mu(1+\mu) + \beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2 \\ +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{11}$$

The mean square acceleration, $\sigma_{a_1}^2$ and $\sigma_{a_2}^2$ of Structures 1 and 2, respectively are expressed as [5]

$$\sigma_{a_1}^2 = \int_{-\infty}^{\infty} \left| x_{a_1}(i\omega) \right|^2 S_0 \, d\omega \tag{12}$$

$$\sigma_{a_2}^2 = \int_{-\infty}^{\infty} \left| x_{a_2}(i\omega) \right|^2 S_0 \, d\omega \tag{13}$$

where $x_{a1}(i\omega)$ and $x_{a2}(i\omega)$ are the harmonic transfer function for acceleration responses, x_{a_1} and x_{a_2} respectively and are expressed by

$$x_{a_1}(i\omega) = \frac{\begin{array}{l}\omega_1^2\omega_2^2 - \omega^2(\Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1} + \omega_1^2) \\ -i\omega^3(\Delta_1) \\ +i\omega(\omega_1^2\Delta_2 + \omega_1^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}}{\begin{array}{l}\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) \\ +\omega_1^2\omega_2^2 - i\omega^3(\Delta_1 + \Delta_2 + \Delta_d) \\ +i\omega(\omega_1^2\Delta_2 + \omega_1^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}} \tag{14}$$

$$x_{a_1}(i\omega) = \frac{\begin{array}{l}\omega_1^2\omega_2^2 - \omega^2(\Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1} + \omega_2^2) \\ -i\omega^3(\Delta_2) \\ +i\omega(\omega_1^2\Delta_2 + \omega_1^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}}{\begin{array}{l}\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) \\ +\omega_1^2\omega_2^2 - i\omega^3(\Delta_1 + \Delta_2 + \Delta_d) \\ +i\omega(\omega_1^2\Delta_2 + \omega_1^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1})\end{array}} \tag{15}$$

The mean square acceleration response can be obtained by integrating Eqs. (12) and (13). Now considering equal damping ratio in both the structures i.e. $\xi_1 = \xi_2 = \xi$, the mean square acceleration, $\sigma_{a_1}^2$ and $\sigma_{a_2}^2$ of Structures 1 and 2, respectively are given as

$$\sigma_{a_1}^2 = 2\pi S_0\omega_1 \left[\frac{\begin{array}{l} \beta(1+\beta)^2\xi((-1+\beta)^2+4\beta\xi^2)(\xi+4\xi^3) \\ +(1+\beta)((1+\beta)\mu(1+4\xi^2)((-1+\beta)^2+8\beta\xi^2) \\ +4\beta^2\xi^2(3-\beta+2\beta^2+4(1+3\beta)\xi^2))\xi_e \\ +4\xi(3\beta^3+\beta^4+\beta^5+4\beta\mu+\beta^2\mu+ \\ 3\beta^3\mu+2\beta^4\mu+\mu^2+3\beta\mu^2+\beta^2\mu^2 \\ \quad +4(\beta+\mu)(\mu+\beta(3\mu+\beta(2+3\beta+\mu)))\xi^2)\xi_e^2 \\ +4(\beta^2+\mu)((1+\mu)(\beta^2+\mu)+4(\beta+\mu)^2\xi^2)\xi_e^3 \end{array}}{\begin{array}{l} 4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2) \\ \quad +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e \\ \quad +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu) \\ \quad +3\beta\mu(1+\mu) \\ \quad +\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2 \\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3) \end{array}} \right] \qquad (16)$$

$$\sigma_{a_2}^2 = 2\pi S_0\omega_1 \left[\frac{\begin{array}{l} \beta^2(1+\beta)^2\xi(1+4\xi^2)((-1+\beta)^2+4\beta\xi^2) \\ +\beta(1+\beta)((-1+\beta)^2\beta(1+\beta) \\ \quad +4(2\mu+\beta(1+\beta+\beta^2+\beta^3-\mu+3\beta\mu))\xi^2 \\ \quad +16\beta(2\beta(1+\beta)+(3+\beta)\mu)\xi^4)\xi_e \\ +4\xi(\beta^3+3\beta^4+\beta^5+2\beta\mu+3\beta^2\mu \\ \quad +\beta^3\mu+4\beta^4\mu+\mu^2+\beta\mu^2+3\beta^2\mu^2 \\ \quad +4\beta(\mu+\beta)(3\mu+\beta(1+\beta(3+\beta) \\ \quad +2\mu))\xi^2)\xi_e^2 \\ +4(\beta^2+\mu)((1+\mu)(\beta^2+\mu)+4(\beta+\mu)^2\xi^2)\xi_e^3 \end{array}}{\begin{array}{l} 4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2) \\ \quad +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e \\ \quad +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu) \\ \quad +3\beta\mu(1+\mu) \\ \quad +\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2 \\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3) \end{array}} \right] \qquad (17)$$

IV. NUMERICAL STUDY

The variation of frequency response function for displacement and absolute acceleration for an undamped and 5% structural damping are shown in Figs. 3 and 4, respectively for different value of ξ_d. The two SDOF connected structures with their mass ratio $\mu = 1$ and frequency ratio $\beta = 2$ are considered implying that the Structure 1 is said to be soft structure and Structure 2 is said to be stiff structure. From the above Figs. it is interesting to note that with the increase in the ξ_d, the value of peak frequency response functions decrease and after reaching certain minimum value it increases with further increase of ξ_d. For low values of ξ_d, the peak value of frequency response function occurs at the natural frequency of the respective structures and it shifts

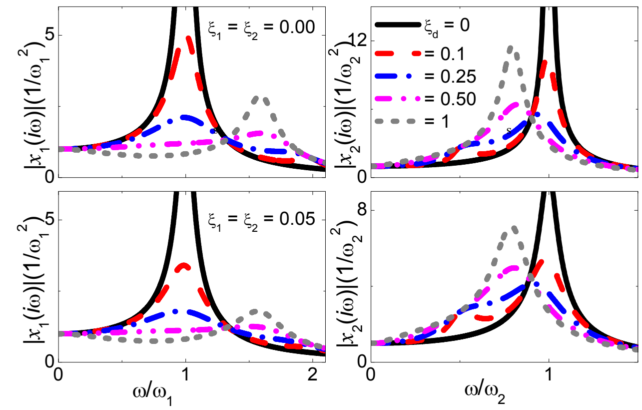

Fig. 3 Variation of frequency response function of displacement against excitation frequency for different damper damping coefficient ($\mu = 1$ $\beta = 2$)

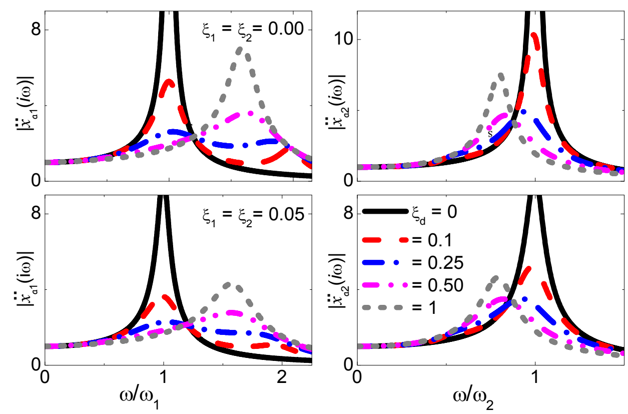

Fig. 4 Variation of frequency response function of acceleration against excitation frequency for different damper damping coefficient ($\mu = 1$ $\beta = 2$)

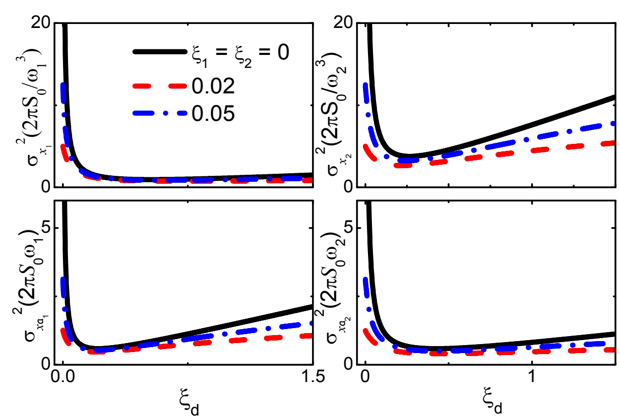

Fig. 5 Variation of mean square response against damping coefficient of damper ($\mu = 1$ $\beta = 2$)

to the combined frequency of the structures for higher values of the ξ_d. This indicates that there is certain value of ξ_d for which the area of the frequency response function will become minimum and yielding the minimum value of the mean square response. The variations of mean square responses of the two structures against the ξ_d are shown in Fig. 5 for different damping ratios of the connected structures (i.e. $\xi_1 = \xi_2 = 0, 0.02$ and 0.05). It is observed from the graph that as ξ_d increase, the mean square responses decreases up to certain value and with further increase in ξ_d the mean square responses increase. There exists an optimum value of ξ_d to yield the mean square responses under stationary white-noise random excitation. Thus, for a given structural system connected with a viscous damper, there exist an optimum value of the damping for which the mean square response of structures attain the minimum value.

V. EFFECT OF SYSTEM PARAMETERS

The variation of optimum damping coefficient of damper and corresponding mean square displacement response against the frequency ratio β for different value of mass ratio (i.e. $\mu = 1, 1.5, 2$) are shown in Fig. 6. It is observe that increase in frequency ratio increases the optimum damping coefficient of damper, where as it decreases the mean square responses. An increase in mass ratio decreases the optimum damping coefficient of damper for both structures. An increase in mass ratio increases the mean square displacement response of the flexible structure, where as it decreases the mean square displacement of stiff structure.

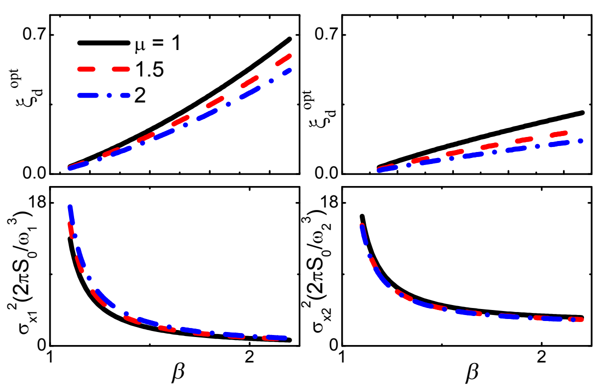

Fig. 6 Effect of frequency ratio and mass ratio on the optimum damping of damper and corresponding mean square displacement response

The variation of optimum damping coefficient and corresponding mean square acceleration response against the frequency ratio β for different value of mass ratio (i.e. $\mu = 1, 1.5, 2$) are shown in Fig. 7. It is observed that increase in frequency ratio increases the optimum damping of damper and decreases the corresponding mean square acceleration responses. An increase in mass ratio decreases the optimum damping of damper. An increase in mass ratio increases the mean square response of the flexible structure where as it decreases the mean square acceleration response of stiff structure.

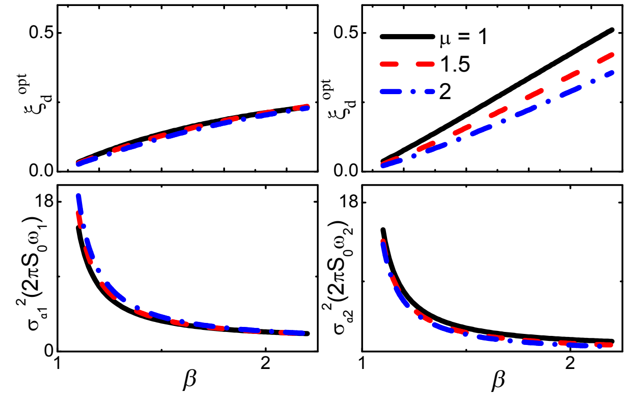

Fig. 7 Effect of frequency ratio and mass ratio on the optimum damping of damper and on corresponding mean square acceleration response

VI. CONCLUSIONS

The dynamic response of two adjacent SDOF structures connected with viscous damper subjected to random excitation is investigated. The effect of system parameters such as frequency ratio and mass ratio on the optimum damping of damper and corresponding mean square response is investigated. From the trends of the results of present study, the following conclusions are drawn.

1 The viscous damper is found quite effective for random response control of adjacent SDOF coupled structures.
2 For a given coupled structural system there exists an optimum damper damping for which the mean square displacement and mean square absolute acceleration of connected structures attains the minimum value.
3 The optimum damping of the viscous damper increases with the increase in the frequency ratio and decrease with the increase in the mass ratio. The corresponding mean square responses at optimum damper damping decreases with the increase in the frequency ratio.
4 The effect of mass ratio on mean square displacement and mean square acceleration responses is marginal.

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